

Chapter 1
Introduction to Numeracy: A *Quantitative* Literacy

*“Don’t say you hate arithmetic,
And find it dull and dry.
You’ll find it most astonishing
If you sincerely try.”*

“Farmer Brown’s boy used to feel that way, but he doesn’t anymore. He never could see any use in puzzling over sums in school. He said that there wasn’t anything in it; nothing but hard work. He used to complain about it at home. Farmer Brown would listen awhile, then he would say, “If you live long enough, my son, you will find that figures talk and they tell the most wonderful things.” There was always a twinkle in his eye when he said this.”

-Thornton W. Burgess
The Adventures of Bob White

Chapter 1: Introduction to Numeracy

This text is essentially about quantitative literacy, or numeracy as it is being called today. The mathematics contained herein stands alone as a foundation for students wishing to actively participate in the increasingly complex, technologically advanced world that we live in. This is a world that is becoming more and more saturated with data and numbers on a daily basis at the level of newspapers, magazines and the evening news. Important societal issues are argued and policy decisions made on the basis of statistical evidence that requires fundamental vocabulary and conceptual knowledge. Ours is a democratic society based on the ideal of equal opportunities for all, but the reality is of course something far less than this. Opportunities exist for those capable of understanding and communicating information, much of which is now quantitative in nature.

A small town in Western New York (to remain nameless) announces a 3% increase in taxes. Phew! What a relief to the financially strapped citizens of this tiny hamlet struggling to make ends meet. The town board was able to make enough cuts in the budget so the taxes only went up from 6% to 9% ... kudos all around until the tax bills came out. A tax bill that was \$4000 previously, a pittance in the poorest county in New York State, was recalculated the next year to be a whopping \$6000! What happened? What didn't happen in our schools is an education that produces numerate citizens. Instead we have an entire country filled with high school and college graduates who cannot parse the difference between a total change of 3 percentage points, from 6% to 9%, and the much different percent change of 50%.

Consider the following table from the publication *Reading at Risk* from the National Endowment for the Arts:

Art Museum Attendance by US Adults, 2002		
	%	Number Attendees (millions)
All Adults	26.5	55
Did not read literature	11.6	13
Read Literature	43.5	42
Light (1-5 books)	34.9	15
Moderate(6-11 books)	47.2	9
Frequent (12-49 books)	53.1	13
Avid (50+ books)	50.9	5

What does the number 43.5 mean in the first column? If you answered 43.5% of US adults attending art museums in 2002 also read literature, you are incorrect! This table is confusing because the number of attendees who read and don't read literature, 13 million and 42 million respectively from the second column, do make up both parts of the whole of US adults attending art museums in 2002, 55 million. This is clearly not the case for the percentages in the first column since they do not add up to 100%. This is a table designed to inform us about the relationship between reading literature and attending art museums; yet it unintentionally misleads us instead! This table will show up in the homework exercises for Chapter 5: Percentages where the true interpretation of 43.5% will be explored.

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Some of you reading this might be unimpressed about the severity of the impact from adults in the United States lacking basic middle school mathematics skills. Surely there can be no real consequences from misleading tables and confusion over percentages? The sub-prime mortgage fiasco of 2007 and ensuing implosion of investment banking in 2008 would lead us to a different conclusion. Millions of Americans lost their homes due to signing off on ultra-risky loans with variable terms difficult to comprehend. The investment bank of Bear Stearns, which had weathered the Great Depression, collapsed under a loss of confidence in its ability to meet debt obligations, a veritable run on the bank. Awash in “collateral debt obligations” and “liquidity puts”, ivy-league bankers found themselves out of their league, with one CEO frankly admitting: “We just didn’t understand the numbers.” Lucky for them the federal government wasn’t about to let the banking system collapse, and with hundreds of billions of dollars of taxpayer money the banks were bailed out, too bad for the unlucky taxpayers whose delinquent mortgages weren’t important enough for such intervention.

In this text we start from the assumption that communication through quantitative information is central not only to mathematics but also to all classrooms, we define numeracy to mean:

The **skill set** necessary to process quantitative information and the **capacity** necessary to critique, reflect upon, and apply it in making decisions.

An early definition of the term “numerate” appeared in a report on math education in Britain (Cockcroft, 1982) as reported in the text *Mathematics and Democracy* (2001):

We would wish the word numerate to imply the possession of two attributes. The first of these is an “at homeness” with numbers and an ability to make use of mathematical skills which enable an individual to cope with the practical demands of everyday life. The second is an ability to have some appreciation and understanding of information which is presented in mathematical terms.

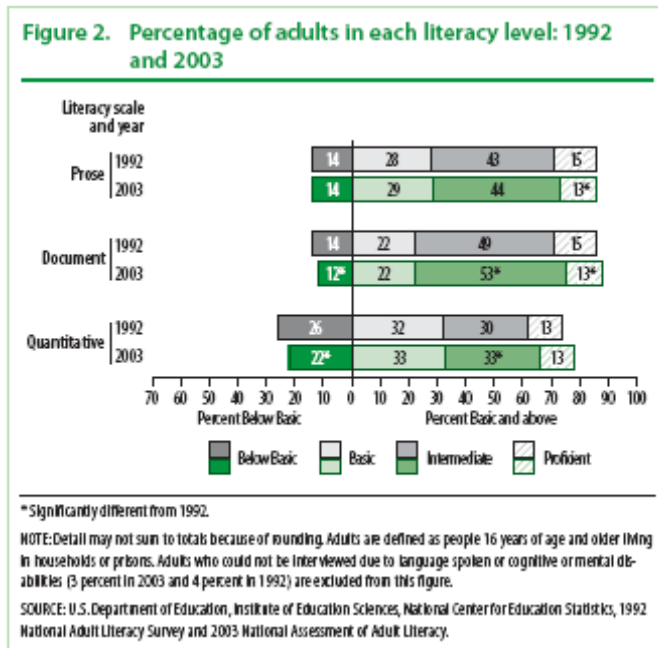
Twenty-five years after this definition the country of Great Britain still struggles with innumeracy as evidenced by a scratch card lottery game called “Cool Cash”, which was pulled from stores due to the public’s inability to deal with negative numbers. The game had a winter theme with a temperature on it; to win you had to scratch off a lower temperature...”Brilliant!” Unfortunately, many of cards had a negative temperature, say -8 degrees, and people would scratch off a -5 degrees and think they had won since 5 is less than 8. The British Government’s Skills for Life survey found that 3 times as many adults in the UK have poor numeracy skills as opposed to poor literacy skills.

In the United States, the National Center for Education Statistics conducted a National Assessment of Adult Literacy (Kutner, Greenberg, Baer, 2005), defining quantitative literacy (QL) as:

The knowledge and skills to perform quantitative tasks (i.e. to identify and perform computations, either alone or sequentially, using numbers embedded in print material).

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This assessment took place in 1992 and 2003, and found that the majority of adults in this country have at best a “basic” mastery of QL, meaning that they are **unable** to “calculate the total cost of ordering specific office supplies from a catalog.” These results highlight the dismaying fact that the vast majority of our population is innumerate, with only 13% considered to be proficient, or able to “compute and compare the cost per ounce of food items”. These statistics remained virtually unchanged from the first assessment in 1992 to the second in 2003:



To be innumerate in today's society is equivalent to being illiterate just 50 years ago. This textbook will guide you through the acquisition of the fundamental **skills** required for numeracy, introducing them in context and providing ample problem sets for you to develop your numeracy **capacity**. The specific numeracy skill set is based on the “proficient” criteria mentioned above: that of being able to compare and contrast the cost per ounce of food items. This is a ratio and thus our skill set begins with this fundamental concept and then introduces all other relevant skills as variations of this theme of ratios.

This approach is unique to this textbook, whereas other textbooks assume knowledge of these topics as a given, we recognize the need to systematically develop and review this fundamental skill set.

By unifying all topics with the common theme of ratios and carefully defining all terms, you will finally understand how everything fits together and how such topics are applied in the real world. The topics are as follows:

- **Ratios and proportions:** A **ratio** is the comparison of the relative size of two quantities; a proportion is the equality of 2 ratios. The ratio of students to computers is 3:2, if there are 90 students how many computers are there?

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- **Proportionality as distinct from proportions:** Quantities are proportional if their **ratio** is constant in a multiplicative sense: doubling one quantity necessitates the other quantity to double also. The miles you drive are proportional to the gas you use but not proportional to the coffee you drink (hopefully!).
- **Constant of proportionality:** The **ratio** of miles to gallons is 100:4, the constant is 25 miles/gallons, computed by scaling the second quantity to one, 25:1.
- **Rates: Ratios** between quantities with different units, and involving the word "per", such as 25 miles per gallon or 1.5 students per computer.
- **Conversions:** A **ratio** signifying a change in units.
- **Scales:** A **ratio** signifying a change in magnitude.
- **Percentages:** A **ratio** where the second quantity is scaled to 100 gives a percentage. The ratio of students to computers is 150 to 100, meaning that the number of students is 150% that of the number of computers. Distinguishing between part-to-part, ratio of boys to girls is 2:3, and part-to-whole, ratio of boys to students is 2:5.
- **Total change vs. percent change:** The number of adults scoring below basic decreased from 1992 to 2003 by 4 percentage points or by 15.4%. Percent change is nothing more than the **ratio** of total change to the original value.
- **Linear models:** The **ratio** of rise to run is the constant called the slope, and can be interpreted as a rate giving total change.
- **Exponential models:** The **ratio** of one output to the previous output is the constant percentage change.
- **Handling data:** Representing quantitative information visually with charts and graphs, numerically with tables, and analytically with equations; and interpreting such mathematical models and drawing inferences from them.
- **Probability:** The **ratio** of the number of ways an event can occur to the total number of possible occurrences.
- **Statistics:** Analyzing data with statistics, measures of central tendency: mean, median, and mode, with attention paid to arithmetic versus geometric means; and measures of spread: range, variation and standard deviation, with standard scores being nothing more than the **ratio** of distance from the mean to the standard deviation.

The urgent need for our students to achieve a basic quantitative literacy, or numeracy, has been the focus of much attention recently, and the rationale goes all the way back to the teachings of John Dewey a century ago that literacy is a great enabler and “the necessary prerequisite for access to a ‘life of widened freedom’”(Orrill, p. xv). Dewey’s concept of a “popular enlightenment” is vital to a successful democracy where “individuals are able to ‘think for themselves’, ‘judge independently’, and discriminate between good and bad information” (Orrill, p. xiv). Lynn Steen (2004) expands Dewey’s ideas to the role of quantitative literacy in today’s world:

the increasing importance of quantitative data for each person’s quality of life and for our collective well-being. We see this not just in terms of jobs and workforce issues but also for everyday issues of personal welfare, social decision making, and the functioning of democratic society (Stein, p. 27).

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The US economy is undergoing its second major shift since the inception of this nation some 230 years ago when it was based on an agrarian way of life supported by local trades-people. The first shift occurred at the turn of the last century with the industrial revolution transforming the farm-based workforce into a manufacturing powerhouse. The dominant manufacturing sector of the 1900's in the US is now giving way to a new sophisticated global economy demanding a whole new arsenal of quantitative skills of our students. Richard Florida, the H. John Heinz III Professor of Regional Economic development, Heinz School of Public Policy and Management, Carnegie Mellon University, in his book, The Rise of the Creative Class (Florida 2002), argues that the dominance of the manufacturing sector in America has given way to a new economic force in the twenty-first century, the Creative Class. In particular, quantitative careers in science, technology, engineering and mathematics comprise a major percentage of what he terms America's Super-Creative Core:

"At the heart of the Creative Class is the Super-Creative Core, comprising 15 million workers, or 12% of the workforce. It is made up of people who work in science and engineering, computers and mathematics, education and the arts, design and entertainment, people who work in directly creative activity, as we have seen" (Florida, 2002).

The primary objective in writing this text is to make students aware that numeracy, like any other literacy is about communication, and that students need to be able to process quantitative information, however it may be presented: visually with charts, numerically in tables, analytically in formulas, or verbally. Students need the skills that allow for critical analysis of such information expressed in varied contexts, and the competence to effectively communicate their knowledge with confidence. Mastery of the topics found in this book will provide students with the vocabulary and essential tools required to maximize their true potential as a member of the ever shrinking global village of the 21st century.

Consider for example, a data set from the National Endowment for the Arts publication, *Reading at Risk*, tracking "literary reading" rates in the US for different age groups:

Literary Reading by Young Adults				
	Percentage of Group			Rate of Decline
Age Group	1982	1992	2002	1982-2002
18-24	59.8	53.3	42.8	-28%
25-34	62.1	54.6	47.7	-23
All Ages	56.9	54	46.7	-18

Figure 1: National Endowment for the Arts, Reading at Risk, Research Division report #46, June 2004, p. xi.

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This table is a wonderful example of quantitative information requiring basic numeracy skills to comprehend. The number 59.8, within the context of this table is actually a sentence: "In 1982 59.8% of the age group of 18-24 year olds in the United States reported that they considered themselves to be literary readers." Next we compare the 59.8 to the number 42.8: "Literary reading rates for this age group have declined from 59.8% to 42.8% over the time period of 1982 to 2002." This sentence introduces the idea of a rate of change, i.e. we now know that the literary reading rates are declining but by how much? Quantifying change is one of the most important uses of mathematics. Calculus itself is nothing more than the "mathematics of change," allowing us to study the dynamic interactions of quantities whose relationships are expressed in terms of functions. The table in figure 1 illustrates two of the most ubiquitous forms of change: total change and percent change. The literary reading rates experienced a total change of 17 percentage points (from 59.8 to 42.8) over the time interval. Total change is very useful for example in computing the amount of money your retirement portfolio increased by from one period to the next; but it does not paint a complete picture of the change in question. If your retirement portfolio undergoes a total change of plus \$10,000 from one year to the next, this could be very good if you started with \$5,000, but would be so-so if you started with \$1,000,000. Percent change allows us to differentiate between these two scenarios, by relating the total change to the initial value: in the first case we increased from \$5,000 to \$15,000, and the total change represents 200% of our initial value of \$5,000:

$$\left(\frac{\text{total change}}{\text{initial}} = \frac{\$10,000}{\$5,000} = 2.00 = 200\% \right).$$

Whereas the second scenario represents only a 1% increase. Getting back to the Literary Reading table, we can now appreciate that the total drop of 17 percentage points from 1982 to 2002 for the 18-24 year olds represents a -28% decline:

$$\left(\frac{-17}{59.8} = -0.284 = -28.4\% \right).$$

Thus the entire 18-24 row from the table constitutes a paragraph, that can only be read by those equipped with the numeracy skills and vocabulary required.

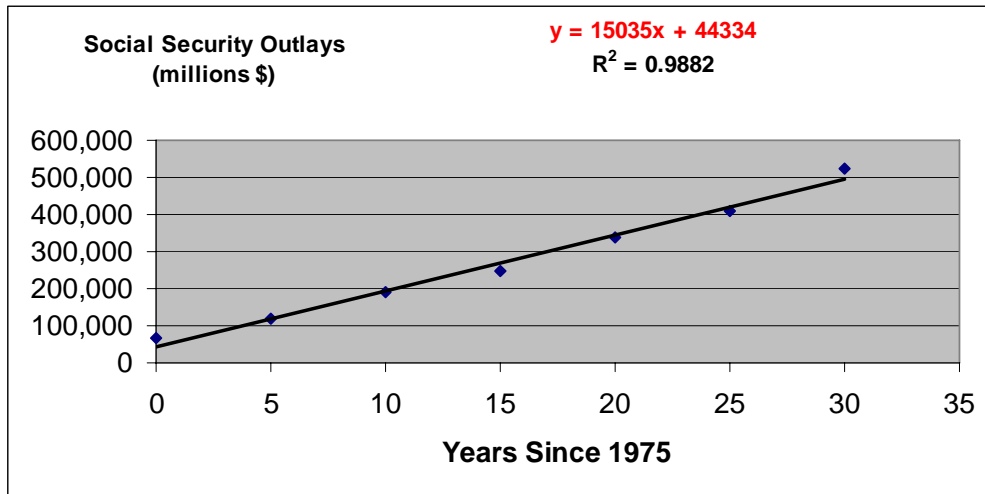
Public policy issues and their related statistics dominate the headlines. Social security in particular has been a major focus of debate, and whether or not this program is in crisis. The following table represents United States budget information taken from the budget report for fiscal year 2002:

Year	Years Since 1975	Social Security Outlays (millions \$)
1975	0	64,658
1980	5	118,547
1985	10	188,623

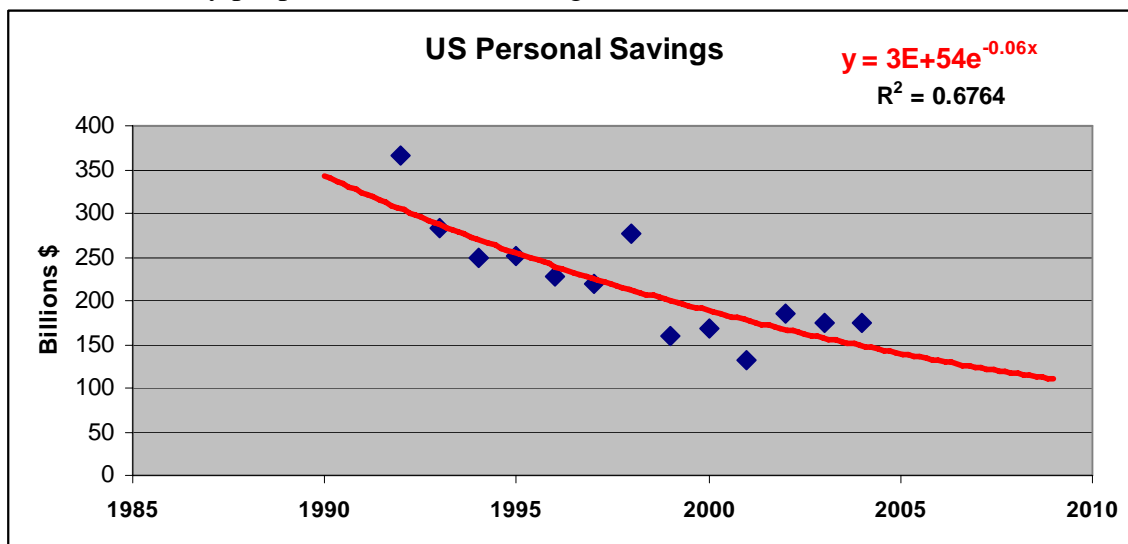
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1990	15	248,623
1995	20	335,846
2000	25	409,423
2005	30	523,305

We can create a chart of the outlays for Social Security using Excel, and add a linear trend-line to this graph:



This visual display of quantitative information is incredibly powerful, and once again requires sophisticated numeracy skills to parse. In particular it is obvious that the Social Security expenses are increasing, but by how much? We can use the equation of the linear trend-line: $y = 15035x + 44334$, to immediately report that the total change is roughly 15 billion dollars per year by interpreting the "slope" of this line. Percent change can be captured using an exponential trend-line for a different data-set representing the amount of money people in the US are saving:



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This graph was created from a data set obtained from the Bureau of Economic Analysis and exemplifies how our media saturated society has become awash in data. The exponential trend-line on this Excel graph includes the very intimidating looking equation: $y = 3E + 54e^{-0.06x}$. Like the linear equation however, it is possible to extract immediate information; in this case the exponent is telling us that the amount of personal savings in the United States has been declining at a continuous rate of 6% per year over the period of 1992 - 2004. This example will be studied in detail in Chapter 7: Exponential Functions.

By working through this text you will be made familiar with the arcane terminology and practices surrounding personal finance: mortgages, annuities, prime rates, tax-deferred retirement accounts, etc. Mastery of this domain will allow you to take charge of your financial future, actively participating in the direction you want your life to take. Too many people are being “taken for a ride” by the multinational lending institutions who prey on their innumeracy. It is interesting to note that Microsoft’s 2003 spell-checker flags the word “numeracy” but recognizes “innumeracy”. A numerate citizen in today’s society does not scoff at the thought of having a million dollars in their retirement account but *expects this* as a matter of course. The numerate citizen understands why Albert Einstein called compound interest the most powerful force in the universe, and knows how to make that power work for themselves in navigating the array of investment options available today. The numerate citizen is interested in legislation regarding Roth IRA’s and the privatization of social security, and actively participates in the democratic process rather than letting special interest groups control the direction of public policy.

The examples above have been selected to demonstrate some of the mathematics to be found in this text, and the technology that we will use, the spreadsheet Excel, to fully explore these concepts. Excel is much more than a data entry tool or glorified calculator. While Excel certainly relieves us from mundane calculations it does force us to enter formulas that require sophisticated numeracy skills to handle, allowing us to capture general situations from individual instances. Excel also provides a highly interactive environment for studying relationships between quantities such as personal savings and years since 1975. These relationships between inputs and outputs are called functions and the function concept is arguably the single most important idea in modern mathematics. As seen above, relationships can be represented verbally in a sentence, numerically in a table, visually in a graph, or analytically with an equation; all of which are equally important in terms of emphasizing different aspects of the underlying function. Excel allows us to see all four representations side by side in an interactive way so that when we change inputs in a numerical table, the outputs and graph immediately change also. This is a powerful means of visualizing the relationships and much more hands-on and effective than the traditional chalkboard emphasis of equations. Excel also has an amazing suite of built-in functions ready to perform more advanced quantitative tasks. Parsing the syntax of these functions and the interplay between the arguments/inputs with the outputs makes this capability of Excel much more than a “black box” tool, but actually requires a high level of understanding on the part of the students if they are to use them correctly. In particular the built-in functions allow us to explore advanced topics in logic, simulations, modeling, optimization, and statistics that otherwise usually require their own separate course to fully develop!

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Recognizing that many schools still use graphing calculators in the classroom due to limited computer lab space, this text also includes guidance on how to use graphing calculator technology in place of spreadsheets. Graphing calculators have evolved along with computer software over the past twenty years, and now include most of the built-in functionality that makes Excel so powerful. An obvious drawback is the tiny black and white display capability of graphing calculators. This text goes beyond the traditional use of the graphing calculator in pre-calculus and calculus classes, with a focus on writing simple programs that emphasize the fundamental input-output relationship of functions and using the sequence mode for exploring recursion. The calculator help sections found in every chapter are written for the TI-84, which was chosen for its dominant role in the marketplace.

All of the mathematics under discussion and found in this text is essentially about quantitative literacy, focusing on critical thinking and problem solving skills. These are skills which will serve all students in all disciplines. General education cannot train every person for every possible situation, but if you learn how to "think" and be adaptable, you will certainly be on your way to a 'life of widened freedom' mentioned at the beginning of his chapter. The inadequate training students in the United States receive in mathematics has been well documented and summarized in the National Commission on Mathematics and Science Teaching's report, *Before It's Too Late* (Glenn 2000). A recent 2003 international study of problem solving skills, *Problem Solving for Tomorrow's World* (PISA 2004), conducted by the Program for International Student Assessment (PISA) and supported by the Organization for Economic Development (OECD) shows that US high school students also score well below average with respect to problem solving. This program lists problem solving skills as being the following:

- The capacity of students to understand problems situated in novel and cross-curricular settings.
- To identify relevant information and constraints.
- To represent possible alternatives or solution paths.
- To develop solution strategies.
- To solve problems and communicate strategies.

These skills are required and developed in virtually every chapter of this text. The unifying theme of ratios provides the framework for exploration and discovery of applying mathematics using technology to solve real world problems. Mathematics is a participation sport and this text is built around the idea of students doing the mathematics as they learn. In class activities take the place of lectures. Homework sets are not mundane repetitions of memorized facts but rather consist of separate stand alone problems in "novel cross curricular settings" with multiple solution strategies possible. Projects replace the standard exam format of most math courses; projects that engage students in the material and challenge them to think creatively.

Enjoy!

Works Cited

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