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Numeracy: A Quantitative Literacy Communicating with Numbers

Chapter Zero: Introduction

An introduction to the rationale for numeracy/quantitative literacy and what is meant by those terms, as well as an overview of the text. Examples from the press involving quantitative information are included to motivate the approach taken in this text. The unifying theme of ratios and the relationship between ratios and the other key concepts developed in the book are introduced here. **This approach is unique to this textbook, whereas other textbooks assume knowledge of these topics as a given, we recognize the need to systematically develop and review this fundamental skill set.**

Chapter One: Introduction to Technology

This chapter was included to introduce the basics of the technology in a simple setting. Students will not have to grapple with numeracy concepts here, just how to use the technology in a rudimentary way. For Excel, the idea of entering data, formatting a spreadsheet, entering a formula, filling a formula, using basic functions, and creating simple graphs are reviewed. For the calculator, it is assumed students will have basic familiarity with graphing calculators in terms of entering equations, graphing, and building tables. Innovative calculator techniques introduced throughout this text will include special instructions. In this chapter, using lists on the calculator are introduced.

1. Creating a spreadsheet template
 - a. Formatting
2. Entering formulas
 - a. Cell References
 - b. Filling formulas
3. Built-in functions
 - a. Sum shortcut button
 - b. Insert function button
4. Using Lists on the Calculator
5. Creating charts in Excel

Chapter Two: An Excel-lent Approach to Relationships

Excel and graphing calculators are the fundamental technology used throughout this text and this chapter provides an **in-depth introduction to spreadsheets** immediate applications via functions. It is assumed that teachers using graphing calculators are familiar with their basic operation, so supplemental calculator information boxes are included in all chapters on innovative ways to use the calculators. This chapter contains a basic introduction to the concept of a mathematical function. However, we elect to use non-standard terminology: “input” (or “input variable”) and “output”. We also strive to break away from the canonical x-and-y notation in favor of variables names that are more suggestive (e.g. “RATE” or “AGE”). Excel is integrated throughout this text to aid in our exploration of quantitative information, allowing us to organize and analyze data, and

freeing us to focus on problem solving while the computer handles mundane computations for us.

This chapter introduces the first **Spotlight on Statistics** which are included at the end of many chapters. In this way the fundamental concepts of statistics are interspersed with the accompanying numeracy topics, providing further context and motivation for learning the mathematics. In this chapter on functions, we introduce the basic descriptive statistics and their equations, which provide excellent practice using Excel to evaluate formulas. The ability to read and use a formula is a critical numeracy skill. The important visual display of data, a histogram, is introduced here as well.

1. Functions
 - a. Function vocabulary
 - b. Wind Chill Example
 - c. Analytic representation i.e. equations
 - d. Input/Output
2. Calculator writing a program
3. Numerical Representations i.e. tables
 - e. Creating tables in Excel
 - f. Filling formulas
 - g. Calculator tables
 - h. Absolute cell references in Excel
4. Graphical Representation
 - i. Graphs in Excel
 - j. Graphs on Calculator
5. Spotlight on Statistics
 - k. Descriptive Statistics: Excel and Calculator
 - l. Histograms: Excel and Calculator

Chapter Three: Ratios and Proportions

This is the first of a three-chapter sequence focusing on the core introductory quantitative literacy topics. We thus start with the definition of the **most fundamental numeracy concept: ratio**. Ratios will be used to motivate all of the other core numeracy topics found in this textbook, unifying seemingly disparate concepts under the simple idea of comparison. The importance of ratio stems from the fact that this concept is a version of the eternal truism: “Everything is relative.” In the real world absolute numbers, such as 43,200 motor vehicle deaths in 2005, lack meaning devoid of context. The absolute number, 43,200, is really quite meaningless by itself, but comparing this number to the total population: 14.57 motor vehicle deaths per 100,000 people, or to the amount of driving: 1.46 motor vehicle deaths per 100 million miles driven, make the number come alive and ‘talk to us’ as Farmer Brown in the opening quotation told us. Ratios and proportions are developed and defined in terms of real world examples. The key idea of a proportional relationship is explored, setting the stage for chapter six on linear functions given the linearity inherent in proportionality. Simple exercises are incorporated to aid in developing students' spreadsheet skills.

The Spotlight on Statistics introduces the important ratios involved with standard scores, such as the Z-score and T-score. These are related to the normal curve which is

also introduced here. The distinction between ratios and fractions is explored in this chapter and exercises, with a particular focus on the ancient Greeks handling of these concepts.

1. Ratios
 - a. Fractions
 - b. Golden ratio
 - c. Set up a proportion
2. Proportionality
 - a. Constant of Proportionality
 - b. Equations
3. Input/Output for ratios
 - a. Using Excel
 - b. Calculator writing a program
4. Spotlight on Statistics
 - a. Z-scores
 - b. Standardized scores
 - c. Bell Curve

Chapter Four: Units, Conversions, Scales and Rates

Units and conversions follow naturally from the previous chapter. Unit-less ratios, when the units for both quantities are identical, are contrasted with ratios involving different units called rates. Conversions are the prototypical example, with the key idea of a scale also being introduced. Rates are used as a real world interpretation of the constant of proportionality from the previous chapter.

The Spotlight on Statistics analyzes Z-scores as unit-less ratios and introduces the concept of the standard error of the mean which is frequently included with data tables. This is related to the normal curve introduced in the last chapter, and interpreted as error related to a confidence intervals.

1. Conversions
 - a. Dimensional analysis/ canceling units
 - b. Translations
2. Scales
 - c. Proportionality
3. Rates
4. Unit-less ratios
5. Spotlight on Statistics
 - d. Standard Error of the Mean
 - e. Distribution of the sample means
 - f. Central Limit Theorem

Chapter Five: Percentages

This is the last of the three-chapter sequence on introductory quantitative literacy topics. Percentages are introduced in terms of ratios with the second quantity being scaled to 100; and interpreted as rates as mentioned in the last chapter. The use of

percentages to quantify change, i.e. percent change, is contrasted with the more intuitive idea of total change. These two fundamental types of growth set the stage for the chapters to follow on linear and exponential functions.

This Spotlight on Statistics relates percentages to percentiles. The important ideas of frequency tables and cumulative frequency are introduced, with the accompanying visual displays as frequency polygons.

1. Percentages
 - a. Interest rates
 - b. Periodic rates
2. Part to Whole ratios
 - c. Part to part ratios
3. Total and Percent Change
4. Growth rates
 - d. Growth factors
5. Percentage points
6. Spotlight on Statistics
 - e. Percentiles
 - f. Frequency polygons

Chapter Six: Linear Functions

This chapter builds on the material developed concerning proportionality and total change. The slope of a line is first introduced as a rate, referring back to examples from Chapters 2 and 3; and from here we develop the equation of a line in the context of examples. Trend-lines using Excel provide real world applications of linear modeling. Regression to the mean and correlation will be fully explored in Chapter 9 so there is no statistical analysis at this time.

1. Linear functions
 - a. Proportionality
2. Calculator scatter plots
3. Slope
 - a. Average change
 - b. Y-intercept
4. Trendlines
 - a. Slopes as rates

Chapter Seven: Exponential Functions

This chapter explores the basics of exponential growth, constant percent change, in contrast to linear growth, constant total change; and builds on the material involving percentages. The key concept of a growth factor is emphasized, and the geometric mean and harmonic mean are contrasted with the arithmetic mean. The irrational number, e , is introduced and used in exponential trend-line equations from Excel. This sets the stage for the next chapter on logarithms which will fully develop these concepts.

1. Compound Interest

- a. Constant percent change
 - b. Growth Factors
- 2. Calculator sequence graphing
- 3. Exponential functions
 - a. Growth rates
- 4. Geometric mean
 - a. Arithmetic/Harmonic means
- 5. Exponential trendlines
 - a. Continuous growth rates
 - b. Calculator exponential modeling

Chapter Eight: Logical Arithmetic

Both growth and decay models are studied through doubling times and half-lives, with logarithms introduced to facilitate in solving for these quantities. The irrational number, e , is now developed as the natural base for exponential functions representing continuous growth. Logarithms are applied to converting between the continuous form and standard exponential equation. Log scales finish this chapter.

- 1. Doubling time
- 2. Logarithms
 - a. Properties
 - b. Logical Arithmetic
- 3. Periodic compound interest
 - a. Monthly/Daily/Continuously
 - b. e
- 4. Natural logarithms
- 5. Log scales

Chapter Nine: Curve Fitting and Correlation

This chapter builds on the modeling introduced in the linear and exponential chapters. We present a non-technical approach to understanding correlation providing another interpretation to the trendlines from modeling. Now we explore the idea of the least-squares best fit and regression to the mean. The correlation coefficient and the coefficient of linear determination are introduced as measures of best fit.

The Spotlight on Statistics provides a more technical analysis of these two coefficients, looking at their formulas and introducing the idea of covariance. An alternative approach to the best fit trendline as regression to the mean is also introduced, providing further insight into the trendlines that Excel generates.

- 1. Oil and Gas
- 2. Line of best fit
 - a. Least squares
- 3. Correlation coefficient
 - a. Coefficient of linear determination
- 4. Spotlight on Statistics
 - a. Covariance

- b. Regression to the mean

Chapter Ten: Models for Growth

This chapter reviews and summarizes the two basic types of growth, linear and exponential. From here the idea of variable growth rates leads to discrete logistic modeling similar to what is found in a differential equations course! The key is Excel's powerful ability to generate data recursively. Using simple reasoning rather than complicated formulas, the student computes data for a wide array of growth patterns.

The Spotlight on Statistics explores the idea of using trendlines to estimate data values. The standard error of the estimate provides a good review of much of the statistics covered so far as well as deepening students' appreciation of modeling. Recursion related to the Fibonacci sequence is explored in the exercises.

1. Linear vs. Exponential growth
 - a. Recursive formulas
 - b. Closed formulas
 - c. Calculator recursive sequences
2. Logistic growth
 - a. Modified growth rates
3. Spotlight on Statistics
 - a. Using trendlines
 - b. Standard error of the estimate

Chapter Eleven: Optimization

We outline the non-calculus, "brute force" method of solving a variety of maximum/minimum problems: use Excel to generate a table of values and/or a graph of the quantity to be optimized then identify the optimal value by inspection. Important economic functions: Demand, Cost, Profit, and Revenue are all introduced, with their associated marginals. These provide examples of quadratic functions.

The Spotlight on Statistics naturally analyzes the least-squares regression line as an optimization problem. The minimum of the sum of the squared distances is found for a specific example illustrating the discrete approach to optimization using Excel.

1. Demand function
 - a. Cost/Revenue/Profit
 - b. Marginal Cost/Revenue/Profit
2. Quadratic functions
3. Max/Min problems
 - a. Brute force technique
4. Spotlight on Statistics
 - a. Trendlines
 - b. Minimizing sum of squares

Chapter Twelve: Financial Health

This chapter introduces the built in financial functions of Excel to study the basics of loans, mortgages, and savings for retirement. In particular, the 3 types of securities:

stocks, bonds, and cash accounts are explored with the aim of developing the financial literacy of the student. The built-in financial functions provide a good review of topics introduced in earlier chapters which used finance for motivating examples.

1. Investments
 - a. Stocks/bonds/mutual funds/cd's
2. Financial functions
 - a. Investing for retirement
 - b. Car loan
3. Amortization schedule
 - a. Calculator financial functions

Chapter Thirteen: Logically!

With the IF statement (and its related logical functions), we explore a number of mathematical problems. The basics of logic terminology are introduced, with a focus on the conditional statement which is compared to Excel's IF-THEN-ELSE format. Applications to business and Game Theory are explored; and the Vlookup function is introduced.

1. Conditional statements
 - a. Converse/Inverse/Contrapositive/Negation
2. IF function in Excel
 - a. Calculator IF-THEN-ELSE programs
 - b. Nested IF functions
 - c. VLOOKUP functions
3. Game Theory
 - a. Prisoner's Dilemma
 - b. Tit for tat

Chapter Fourteen: Random Events and Simulations

We use Excel's random number generator to simulate probabilistic experiments of increasing complexity, then calculate probabilities from these simulations. The Logic functions from the previous chapter are used here to develop relatively sophisticated simulations. Starting with coin tossing and dice rolling, we ultimately progress to waiting lines and queuing theory. This chapter goes hand in hand with the final chapter on statistics.

1. Probability
 - a. Odds
 - b. RAND function
2. Natural in Craps
 - a. Rolling dice
 - b. Random simulations
3. Conditional Probability
 - a. False positives
 - b. Proportionality principle

Chapter Fifteen: Statistics

This final chapter will provide you with an introduction to statistics, one of the most powerful and useful branches of mathematics. The goal here is to wrap up all the spotlights on statistics from prior chapters and whet your appetite enough to motivate any interested people to take a full course in statistics (or two!). Like the other numeracy topics covered in this book there are basic statistical concepts and statistical terminology that is considered to be general knowledge. This chapter reviews the basic measures of center and variation, and then motivates the normal distribution via the binomial distribution using the techniques from the previous chapter.

1. Distribution of a sample data set
 - a. Descriptive statistics
2. Normal distribution
 - a. Binomial distribution
 - b. Properties
 - c. Standard normal distribution
3. Non-normal distributions
 - a. Chebyshev's Theorem
 - b. Central Limit Theorem
4. Standard error of the mean
5. Statistically significant

Appendix: Excel Basics

- A.1 Getting Started
- A.2 Functions and the Fill operation
- A.3 Relative vs. Absolute Cell Addressing
- A.4 Graphing Functions
- A.5 Logical Functions

Appendix: Graphing Calculator Basics

- B.1 Getting Started
- B.2 Functions and Lists
- B.3 Graphing

Appendix: Probability

- C.1 Fundamentals of counting
- C.2 Laws of probability